DIFFUSION OF BUBBLES IN TURBULENT FLOW

V. S. Fedotovskii and V. P. Bobkov

UDC 532.529.6

The diffusion of bubbles in turbulent flow is considered. A relation is found for the coefficients of turbulent diffusion of bubbles and liquid particles of a carrier medium.

When studying the motion of gas–liquid flows, in which the gas phase is present in the form of bubbles, problems of diffusion of the bubbles in turbulent flow may arise. Since the density of the gas bubbles is different from the density of the carrier medium, it would be expected that the pulsating motion of the bubbles and, consequently, also their diffusion in turbulent flow differ from the pulsating motion and diffusion of liquid particles of the carrier medium.

A number of authors [1-3], investigating the motion and diffusion of particles in a turbulent medium, have considered cases of motion of particles whose density is less than the density of the medium. It was shown that the pulsating velocities of these particles (bubbles) are greater than the pulsating velocities of the carrier medium, but an analysis of the diffusion process of these particles has not been undertaken.

In this present paper, we consider the ratio of the coefficients of diffusion of bubbles and liquid particles of the carrier medium in turbulent flow with certain simplifying assumptions. The case of low bubble concentration is considered, in order that the effect of the bubbles on one another and on the turbulent flow characteristics could be neglected. With this assumption, it is sufficient to consider the motion of a single bubble.

Suppose that an element of liquid, containing bubbles of gas, is moving with a velocity u(t). In this case, an expelling force $V\rho du/dt$ acts on the bubbles, by the action of which a relative motion of the bubble with a velocity w(t) results. The equation of motion of the bubble will have the form

$$V\left(\rho_0+\frac{1}{2}\rho\right)\frac{dw}{dt}=V\rho\frac{du}{dt}-F. \quad (1)$$

Here $V(\rho_0+1/2\rho)$ is the sum of the intrinsic and additional masses of the bubble; $w = v - u$ is the relative velocity; and F is the drag force resulting from the relative motion.

In the case of nonsteady motion of a sphere, an integral term (Basset force) occurs in the expression for the drag force, which takes into account the nonsteadiness of the motion; however, because of the mobility of the boundary of the bubble, the vorticity in the boundary layer is far less than in the case of motion of a solid sphere, and the Basset force obviously can be neglected. For a bubble of spherical shape moving in a pure (without surface-active agents) liquid, the expression for the drag force [4]

$$F=12\pi\nu\rho a w \quad (2)$$

coincides well with the experimental data over a wide range of Reynolds numbers, determined by the size of the bubble and the velocity of motion [5]. Taking Eq. (2) into consideration, the equation of motion can be written in the form

$$\frac{dw}{dt}+A\gamma w=\gamma\frac{du}{dt} \quad (3)$$

or, for the absolute velocity of the bubble,

$$\frac{dv}{dt}+A\gamma v=(\gamma-1)\frac{du}{dt}+A\gamma u, \quad (4)$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 31, No. 4, pp. 678-683, October, 1976. Original article submitted August 19, 1975.

where

$$A = \frac{9\nu}{a^2} \quad \text{and} \quad \gamma = \frac{2\rho}{2\rho_0 + \rho} .$$

Considering further the motion of the bubbles in turbulent flow, we note that the equations of motion should contain a term of the type $\nu d^2u_i/dx_k^2$, the contribution of which is significant only for elements of liquid whose dimensions are of the order of a microscale of turbulence. Since as the process of diffusion is determined by large-scale turbulence, this term can be neglected.

Thus, in order to determine the statistical properties of the turbulent velocity pulsations of the bubbles, we shall use the linear Eqs. (3) and (4), in which u, v, and w are random functions. In order to determine the amplitude–frequency characteristics which relate the spectral characteristics of the velocity pulsations of the bubbles and of the medium, we assign a velocity for the medium in the form of a harmonic component, and from Eqs. (3) and (4) we obtain the corresponding solutions for v(t) and w(t). Squaring v(t) and w(t) and averaging with respect to time, we obtain the squares of the moduli of the amplitude–frequency characteristics:

$$|f(\omega)|^2 = \frac{(\gamma + 1)^2\omega^2 + (A\gamma)^2}{\omega^2 + (A\gamma)^2} \tag{5}$$

for the absolute action of the bubbles and

$$|f_0(\omega)|^2 = \frac{\gamma^2\omega^2}{\omega^2 + (A\gamma)^2} \tag{6}$$

for the relative motion of the bubbles. We shall assume further that the turbulence of the carrier flow is characterized by the Lagrange time correlation

$$R(\tau) = \exp\left(-\frac{\tau}{T}\right) . \tag{7}$$

The corresponding expression for the spectral density, defined as the Fourier cosine-transform of the correlation function R(τ), has the form

$$E(\omega) = \frac{2\sigma_u^2}{\pi}\int_0^\infty R(\tau)\cos\omega\tau d\tau = \frac{2\sigma_u^2}{\pi T\left(\frac{1}{T^2} + \omega^2\right)} . \tag{8}$$

The expression for the spectral density of the velocity pulsations of the bubbles relative to a stationary system of coordinates is obtained as the product of the spectral density E (ω) and the square of the modulus of the amplitude–frequency characteristics (5):

$$E^*(\omega) = \frac{2\sigma_u^2[(\gamma + 1)^2\omega^2 + (A\gamma)^2]}{\pi T\left(\frac{1}{T^2} + \omega^2\right)\left[\omega^2 + (A\gamma)^2\right]} . \tag{9}$$

Then the ratio of the dispersions of the velocity pulsations of the bubbles and of the medium will have the form

$$\frac{\sigma_v^2}{\sigma_u^2} = \frac{1}{\sigma_u^2}\int_0^\infty E^*(\omega)\,d\omega = \frac{(\gamma + 1)^2 + A\gamma T}{1 + A\gamma T}. \tag{10}$$

It can be seen from formula (10) that in a liquid with a vanishingly small viscosity ($A \to 0$) the ratio of the dispersions is equal to $(\gamma + 1)^2$. In the other limiting case of a large viscosity of the medium or bubbles of very small dimensions ($A \to \infty$), this ratio tends to unity. Similarly to Eq. (10), we obtain for the dispersion of the pulsations of the relative velocity

$$\frac{\sigma_w^2}{\sigma_u^2} = \frac{1}{\sigma_u^2}\int_0^\infty E_0^*(\omega)\,d\omega = \frac{\gamma^2}{1 + A\gamma T} \tag{11}$$

where

$$E_0^*(\omega) = E(\omega)\,|f_0(\omega)|^2.$$

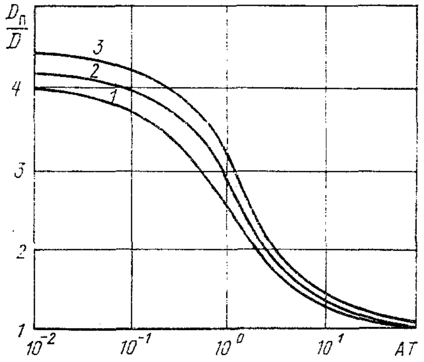

Fig. 1. Dependence of ratio of the turbulent diffusion coefficients of the bubbles and of the liquid particles of the carrier medium on: 1) $\gamma = 1$; 2) 1.5; 3) 2.

By analogy with the coefficient of turbulent diffusion in single-phase turbulence, defined by Taylor as the product of the intensity of the velocity pulsations and the Lagrange scale of turbulence, the coefficient of diffusion of the bubbles can be written in the form

$$D_n = \sigma_v \Lambda^* = \sigma_v^2 T^*, \tag{12}$$

where $T^* = \Lambda^*/\sigma_v$ is the Lagrange time scale of motion of the bubbles.

In the case of motion of bubbles in turbulent flow, the diffusion process at large times is determined by the vortices, the dimensions of which are equal to the integral scale of turbulence. But although the exponential form of the correlation function (7) indicates a statistical relation of the velocity pulsations of all scales of turbulence, it will be assumed that the vortices whose dimensions are equal to the integral scale are statistically independent. Then the Lagrange time scale of the motion of the bubbles is equal to their time of stay in the vortex of the integral scale Λ for the condition that after this time the direction of motion of the vortex is essentially unchanged. Thus, we obtain

$$T^* = \frac{\Lambda}{\sigma_w}, \tag{13}$$

where

$$\Lambda = \sigma_u T. \tag{14}$$

Substituting Eq. (11) in (13) and taking account of Eq. (14), we obtain the ratio of the Lagrange time scales of motion of the bubbles and of the medium:

$$\frac{T^*}{T} = \frac{\sqrt{1 + A\gamma T}}{\gamma}. \tag{15}$$

It can be seen from formula (15) that if $AT > (\gamma^2 - 1)/\gamma$, then the time of stay of a bubble in the vortex will be greater than the time during which the vortex is moving predominantly in one direction. In this case, the bubbles will perform random wanderings inside large-scale vortices, transported by the latter along the flow.

Substituting Eqs. (10) and (15) in Eq. (12), we obtain the ratio of the diffusion coefficients of the bubbles and of the carrier medium for $AT < (\gamma^2 - 1)/\gamma$:

$$\frac{D_n}{D} = \frac{\sigma_v^2 T^*}{\sigma_u^2 T} = \frac{(\gamma + 1)^2 + A\gamma T}{\gamma\sqrt{1 + A\gamma T}}. \tag{16}$$

When $AT > (\gamma^2 - 1)/\gamma$, assuming for the time scale of the motion of the bubbles the Lagrange time scale of motion of the medium T, we obtain

$$\frac{D_n}{D} = \frac{\sigma_v^2}{\sigma_u^2} = \frac{(\gamma + 1)^2 + A\gamma T}{1 + A\gamma T}. \tag{17}$$

It can be seen from Eqs. (16) and (17) that the ratio of the diffusion coefficients of the bubbles and of the carrier medium is greater than unity. In the limiting cases $AT \to 0$ and $AT \to \infty$, the ratio of the diffusion coefficients is equal to $(\gamma + 1)^2/\gamma$ and 1, respectively, (Fig. 1).

Formulas (10), (11), (16), and (17) are obtained for the case of the motion of bubbles under the action of turbulent velocity pulsations of the carrier medium of all scales [integration in Eqs. (10) and (11) was carried out from zero to infinity]. However, the motion of spherical bubbles, whose dimensions considerably exceed the microscale of turbulence, will be determined only by those turbulent velocity pulsations whose scale is greater than the dimensions of the bubble. Therefore, we shall determine the dispersions of the absolute and relative velocity pulsations in this case as

$$\frac{\sigma_v^2}{\sigma_u^2} = \frac{1}{\sigma_u^2} \int_0^{\omega_0} E^*(\omega)\, d\omega \tag{18}$$

and

$$\frac{\sigma_w^2}{\sigma_u^2} = \frac{1}{\sigma_u^2} \int_0^{\omega_0} E_0^*(\omega)\, d\omega. \tag{19}$$

Here the upper limit of integration ω_0 is the characteristic frequency of the velocity pulsations of the liquid, the scale of which is equal to the size of the bubble, i.e.,

$$\omega_0 \sim \frac{u_\lambda}{\lambda} = \left(\frac{\varepsilon}{\lambda^2 \rho}\right)^{\frac{1}{3}}, \tag{20}$$

where u_λ is the change of the pulsation velocity at a distance λ [4]; and ε is the rate of energy dissipation per unit volume, expressed in terms of the work per unit volume as [6]

$$\varepsilon = 1.65\, \rho \frac{\sigma_u^3}{\Lambda}. \tag{21}$$

Substituting Eq. (21) into Eq. (20), we obtain

$$\omega_0 = \left(\frac{1.65\sigma_u^2}{Ta^2}\right)^{\frac{1}{3}}, \tag{22}$$

while from relations (18), (19) and (22), using Eqs. (13) and (14), an expression can be obtained for the diffusion coefficients D_n/D.

If the dimensions of the bubbles are such that the characteristic frequency of the pulsations ω_0 is one order from the characteristic turbulence frequency $1/T$, and the quantity A is much less than these quantities, then the expressions for the spectra of the absolute and relative velocity pulsations of the bubbles are simplified:

$$E^*(\omega) \approx \frac{2(\gamma + 1)^2 \sigma_u^2}{\pi T\left(\frac{1}{T^2} + \omega^2\right)} \tag{23}$$

and

$$E_0^*(\omega) \approx \frac{2\gamma^2 \sigma_u^2}{\pi T\left(\frac{1}{T^2} + \omega^2\right)}. \tag{24}$$

Integrating Eqs. (23) and (24) from zero to ω_0, we obtain

$$\frac{\sigma_v^2}{\sigma_u^2} = \frac{2(\gamma + 1)^2}{\pi} \operatorname{arctg} \omega_0 T \tag{25}$$

and

$$\frac{\sigma_w^2}{\sigma_u^2} = \frac{2\gamma^2}{\pi} \operatorname{arctg} \omega_0 T. \tag{26}$$

Determining from Eq. (26) the time scale of the motion of the bubbles according to Eq. (13) and taking account of Eqs. (22) and (14), we obtain the ratio of the diffusion coefficients for $AT < (\gamma^2 - 1)/\gamma$ in the form

$$\frac{D_n}{D} = \sqrt{\frac{2}{\pi}} \cdot \frac{(\gamma + 1)^2}{\gamma} \left[\operatorname{arctg} 1{,}18 \left(\frac{\Lambda}{a} \right)^{2/3} \right]^{1/2}. \tag{27}$$

We note that with a reduction of the dimensions of the bubbles, the ratio of the diffusion coefficients D_n/D tends to $(\gamma + 1)^2/\gamma$ just the same as according to formula (16).

Thus, the diffusion coefficient of the bubbles for defined conditions can considerably exceed the turbulent coefficient of the liquid particles of the carrier flow. If the density of the gas in the bubble is much less than the density of the liquid, then the maximum value of the ratio of the diffusion coefficients amounts to 4.5 as $AT \to 0$.

NOTATION

$u(t)$, $v(t)$, $w(t)$	are the velocities of medium and bubbles (absolute and relative);
σ_u, σ_v, σ_w	are the intensities of the velocity pulsations of the medium and bubbles (absolute and relative);
ν	is the coefficient of kinematic viscosity of the medium;
a	is the radius of the bubble;
ρ, ρ_0	are the densities of the medium and of the gas in the bubbles;
$f(\omega)$, $f_0(\omega)$	are the amplitude–frequency characteristics for the absolute and relative motions of the bubbles;
E, E^*, E_0^*	are the spectral densities of the velocity pulsations of the medium and of the bubbles (absolute and relative);
T, T^*	are the Lagrange time scales of motion of the medium and of the bubbles;
Λ, Λ^*	are the integral scales of motion of the medium and of the bubbles;
D_n, D	are the turbulent diffusion coefficients of the bubbles and liquid particles of the carrier medium;
ω	is the angular frequency;
V	is the volume of a bubble.

LITERATURE CITED

1. D. S. Azbel' and Z. G. Narozhenko, Theoretical Principles of Chemical Technology [in Russian], No. 6 (1969).
2. J. O. Hinze, Turbulence, McGraw-Hill (1959).
3. G. K. Batchelor, in: Mekhanika [Periodic Collection of Translations of Foreign Articles], No. 3 (1968), p. 108.
4. V. G. Levich, Physicochemical Hydrodynamics [in Russian], Fizmatgiz, Moscow (1959).
5. G. K. Batchelor, Introduction to Fluid Dynamics, Cambridge University Press (1967).
6. T. Von Karman, Proc. Roy. Soc., 164A, 192 (1938).